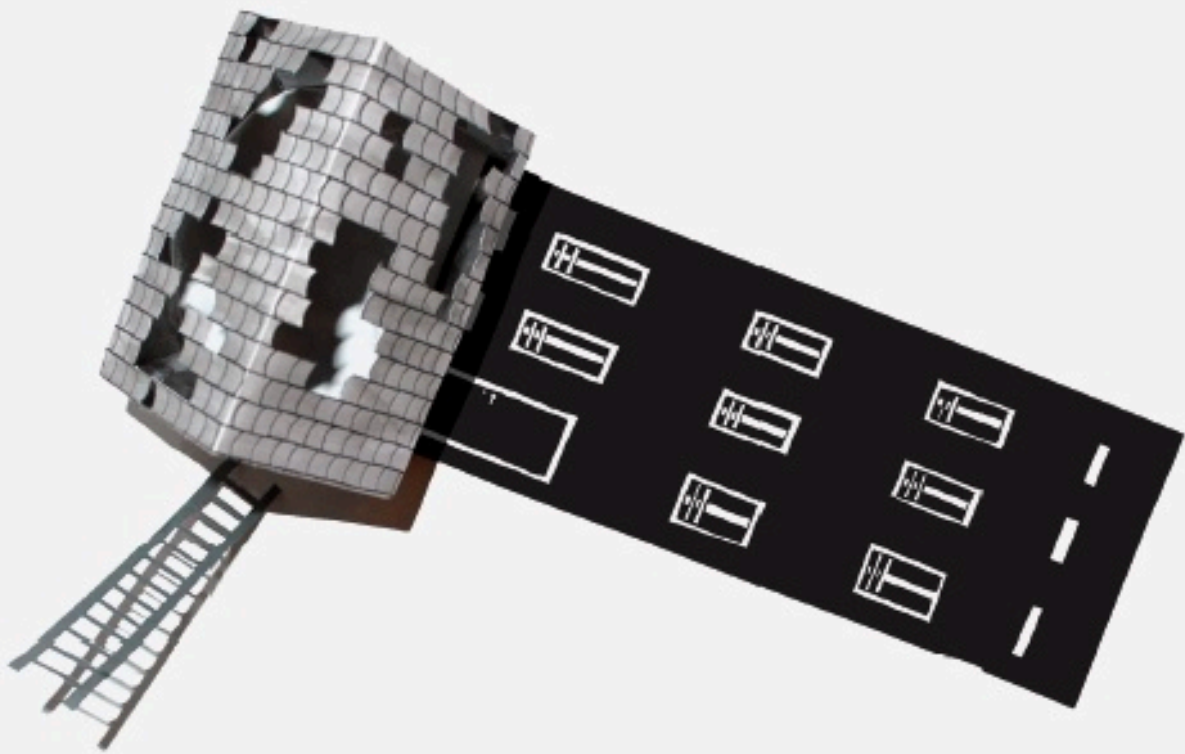


Passed over in Silence

On Wittgenstein's
Tractatus and its System



Jaap van der Does

Studies in Logic

Volume 28

Passed over in Silence
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and its System

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Passed over in Silence. On Wittgenstein's *Tractatus* and its System
Jaap van der Does

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I like to have time for the things I do. I think that we're rushing too much nowadays. That's why people are nervous and unhappy – with their lives and with themselves. How can you do anything perfect under such conditions? Perfection takes time.

Marilyn Monroe

Ring the bells that still can ring
Forget your perfect offering
There is a crack in everything
That's how the light gets in.

Leonard Cohen

For Marjon, Ellis, Thomas to whom I belong
in this or any other order. . .

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Introduction

In philosophy, seen from a distance, there are authors and analysts. An author holds that true insight requires the beauty of everyday language. By contrast an analyst is quite indifferent to the force of a well-chosen phrase. In his opinion understanding only results from charting the logical structure of a problem in minute detail. Authors think such technicalities distracting and irrelevant. On this view: Sartre is an author, Carnap an analyst.

It is comforting at times to invent labels, but ‘labels are for the things men make, not for men’. So if you label anyway, ‘don’t glue them on, and have replacements handy’. (Rex Stout, motto of Themerson (1974).) Early Wittgenstein, in particular, was an author who started his philosophical activity with a passion of the analyst: the nature of propositions and logic.

Due to the unusual combination of art and analysis, Wittgenstein’s *Tractatus Logico-Philosophicus* was likely to attract a wide variety of readers. Those labeled at one extreme take the text as a logical poem with some Russellian formulas interspersed as ready-mades. Such readers leave the suggestion as is that major parts of the book are about a logical system, based on genuine logical insights, and feel justified in doing so due to the author’s remarks that in the end all is nonsense. The readers labeled at the other extreme are impressed by the text’s stark formulation, but still think there is no interest in contemplating the booklet for long. Some of its basic ideas on logic and semantics are also found by others, such as truth tables by Frege, Pierce, or Post, who present them in a less vague, more profound way. And obviously the sum of literature and logic is nonsense.¹

I find myself keeping maximal distance from the readers as labeled at both extremes. There is no royal road to nonsense of the tractarian variety, because it is based on the logical insight that the system of descriptive language is not reflexive. This insight can only be grasped after a full study of the tractarian system and how it relates to Russell’s paradox. Indeed, I hold that despite its artistic, somewhat paradoxical presentation, the system allows for a detailed interpretation, and that such an interpretation is helpful for readers with less firm logical intuitions as young Ludwig happened to have. Spelling out the system’s particulars is gratifying as it helps making sense

¹As to the one extreme, *The Times* obituary of Wittgenstein described the *Tractatus* as a logical poem (Edmonds and Eidinow, 2001, 228). As to the other extreme, quite a few logicians and mathematicians would be examples; e.g. Menger (1994), 89.

of statements that at first were fascinating but hard to follow. From a social point of view, the task may well be ungrateful – the result is likely to be too philosophical for the logician, and too logical for the philosopher, – but I cannot resist the temptation of trying to understand the *Tractatus* in this way.

Aim

This book sets out to show that in spite of its condensed literary presentation the *Tractatus* has a coherent reading, both philosophically and technically. It takes the *Tractatus* as an ethical deed, and a primary aim of the book is to show how Wittgenstein's ethics is related to his highly original 'symbolic turn'.² It is without doubt that human life poses a problem, and that a way of living has to be found to resolve it. The *Tractatus* strongly suggests that insight into the world's contingency via its close ties with meaningful language would pave the way toward the proper way of living.

In line with the *Tractatus*, the book mainly offers a detailed overview of the symbolic turn. The symbolic nature of contingent propositions in logical space is charted in detail, while logical propositions are characterized as empty forms about nothing. To this end, logical space, object, object-form, identity of object-form, state of things, picture, projection, proposition, sign, symbol, situation, sense, truth, logical consequences and their formal relationships are made as explicit as possible; sometimes even in the form of proofs. Yet, the main purpose is to attain clarity, not so much logical depth.

A prime advantage of the current approach is that it shows early Wittgenstein to have been sufficiently precise concerning his so-called perfect notation; be it, as always, without specifying it in much technical detail. The idea of a perfect notation allows for some alternatives, but minimally it captures the symbol that a proposition expresses, which is the contingent nucleus of its sense disregarding logical parts. In a perfect notation the representation of $(p \wedge q) \vee (p \wedge \neg q)$ is the same as that of p . Thus, a perfect notation ensures that equisignificant propositions have the same form and content. Insofar as the formalities of this core aspect of the *Tractatus* have received attention, it is assumed that Wittgenstein abandoned his quest for perfection because it is logically impossible. See e.g. Potter (2009), section 24.4. By contrast, I think one should distinguish between aiming for a perfect notation of sense, and aiming for a transparent notation to show logic's triviality. The latter is indeed impossible due to the undecidability of the logic. If Wittgenstein had paid sufficient attention to his perfect notation of sense for the infinite case, he would have noticed immediately that logic, although still *about* nothing, is not trivial at all.

Apart from treating of the relationship between ethics and the symbolic

²In 1967, Paul Engelmann's *Letters from Ludwig Wittgenstein* has published with an editor's appendix of Brian McGuinness (Engelmann, 1967). Engelmann was the architect of Margaret Stonborough's house at the *Kundmangasse* until Wittgenstein took over. He was also one of the first to publish on the ethical impact of the *Tractatus*. McGuinness' appendix has the famous quotation from Wittgenstein's letter to Von Ficker that made Wittgenstein's ethical intent certain; see page 2. Janik and Toulmin (1973) present this view on the *Tractatus* in a wider cultural-historic setting. 'Symbolic turn' is Michael Potter's apt term. See his history of ideas *Wittgenstein's Notes on Logic*.

turn, the current approach also gives insight into how objects compare with typed-entities; into the nature of signs in logical space; into the different ways in which the notion of projection can be interpreted; into the nature of truth-operations and how they compare with truth-functions; into the notion of sense for contingent and logical propositions; into the niceties of logical consequence; into substitution in a representation of sense; into how the tractarian system solves Russell's paradox. Not only the finite system is covered but also a natural infinite generalization that Wittgenstein's text just hints at. It is shown that Wittgenstein's use of truth-table and graphical signs anticipates the elegant tableau methods developed much later. Despite the bad press Wittgenstein's treatment of quantification received round about 1980, it is here argued to be correct.

Philosophical engineering

Understanding Wittgenstein's early philosophy seems to require a minute study of the tractarian system – often called 'the system' here, – but as far as I know a precise model that does justice to the entire text has never been defined. Why? Wittgenstein presents the system as a philosopher and writer, who would have been distracted by too much attention for detail. Primarily concerned with general philosophical ideas, he sketches the outlines of the system in a lucid style as its features come to his attention. One would presume that this gives enough information to enable philosophical engineers to fill out the details, in the non-literary, dry fashion, but few felt the urge to take up this rôle. (When working on his *hausgewordene Logik* – logic turned into a house, a term from the *Familienerinnerungen* of his sister Hermine Wittgenstein, – Wittgenstein assumed a similar rôle himself. Cf. McGuinness (2001), 21.) Roundabout 1930 Waismann seemed to have suggested writing an introductory book on the *Tractatus* enriched with some newer insights, but by then Wittgenstein saw too many shortcomings in his system to find it useful. See part IV of McGuinness' introduction to Wittgenstein (1984).

In this book I will mainly act as a philosophical engineer, working on the system's design as it is specified in the text. I will try to keep the philosophical debate concise, and only engage in it if I have to argue for a particular reading. The secondary literature of the *Tractatus* has grown into a *mer à boire*, which offers some beautiful vistas, but at the same time evaporates any ambition to concern oneself with more than a specific aspect of the philosophy and its the system. By contrast, to detail the system almost no thesis can be left out of consideration. To keep this manageable I should try to find an optimum between being clear and being brief.

An ethical deed

Philosophical engineering may leave the impression that the *Tractatus* is chiefly concerned with logico-linguistics, like on its reception by the logical positivists in the early twentieth century. To prevent this from happening, I stress its ethical concern from the outset. In a way this book is a technical sequel to Martin Stokhof's 'World and Life as One' (Stokhof (2002)). Even

when his book was still in manuscript, I was particularly impressed by his view on the relation between ethics and ontology in Wittgenstein's early thought. Still, I felt that a more detailed presentation of the system would be helpful.

Advantages

Formalizing the system is not an aim in itself but has a few advantages.

An improved insight into the system of the *Tractatus* will lead to a better understanding of how ethics and the symbolic turn are related to each other. In this regard, a single, coherent model of the system amounts to giving a strong interpretation of the text that can be criticized more effectively.

A rather formal interpretation of the system allows one to gauge accurately how Wittgenstein's approach to logic and sense compares with and deviates from that of Frege and Russell. It makes clear how innovative Wittgenstein's ideas on the perfect notation are, in which the ideal representations of sense are logical structures that mirror possibilities in an intensional domain. It is also a little therapeutic to arrive at such insights: one has to unlearn modern logic to a certain extent to see the specifics of Wittgenstein's approach.

Studying the tractarian system makes one fully aware that logic and semantics are not philosophically neutral: a proper understanding of their basic concepts is at the heart of any philosophy of logic and language. E.g., there are crucial differences between Frege's and Russell's 'platonic' views on the one hand – with a third realm providing the objects and structure that logic is about, – and early Wittgenstein's 'aristotelean' view on the other – where language is a worldly matter concerning the structure and content of reality, pictured within logic's empty frame.

Insight in the early system gives a strong foothold when studying Wittgenstein's critique in his later works, namely: that not all elementary propositions are independent of each other and so that not all logic is truth-functional; that quantifiers should not be reduced to truth-functions; that the idea of absolutely simple names is based on confusing the meaning of a name with its bearer; that there are no absolute notions of simplicity and complexity that analysis should unveil; that language lacks a unique system of representation capturing the essentials of sense. A perspicuous view on the early system may allow one to trace what can be retained as one of the more modern outskirts of language, which should no longer be mistaken dogmatically for *the* system showing its semantic essence.

A full interpretation of the system should contribute to improving the quality of philosophical discussion. It naturally leads to the idea of ostensive philosophy, which allows for a strong alternative to so-called resolute readings of the *Tractatus*.³

³In this book I have little to say about resolute readings. In a way resolute readings of the *Tractatus* are prime examples of weak interpretation. They postulate a few theses to be 'frame remarks', which are the only theses that should be taken seriously. The frame remarks are then used to argue that the *Tractatus* and its early sources offer a deflationary philosophy of mere nonsense. To come to such a reading much evidence to the contrary must be resolutely ignored. See Hacker (2000) and Proops (2001b) for details.

Cloud and drop

For a text that aims to concentrate on the system, some paragraphs may still appear rather verbose. But besides presenting a more formal framework, I want to make transparent how the details given relate to the text. Also, there are passages where the brevity of the *Tractatus* requires me to argue for a particular reading on which the formalization is based. Freely paraphrasing later Wittgenstein: I cannot avoid showing how some philosophical or philological clouds are condensed into a drop of logic. That I present both cloud and drop is based on a principle of ethics: help your opponents to find a proper attack.

Attempting the impossible?

Some readers will observe that every now and then I attempt to say what according to early Wittgenstein cannot be said. This observation is not always correct. For instance, specifying a formal concept as an abstraction over logical space, e.g., the form of an object or the identity of such forms, is part of the system (4.122). Yet there are other areas where the observation must be granted. But here I find myself in good company, for of course early Wittgenstein accused himself of doing exactly the same.

If required, there are two ways to argue in favor of my approach. Firstly, before criticizing a philosophy one should make a serious attempt to grasp it. Without literary aspiration, my reconstruction of the tractarian system mainly uses the methods of the analyst. This approach may help some to get a better view on early Wittgenstein's ostensive philosophy – to show the limits of what can be said, – than his more artistic presentation. Secondly, when studying a system the methods used must leave it intact, but they do not have to comply with all of its principles. Whenever required I do step back and employ methods that from a tractarian perspective are way out of line. E.g. they use non-tractarian notions, or compare the system with other systems. Without being too finicky about it, I have labeled such parts as 'reflections'.

Overview

The structure of this book is concentric. Chapter 1 starts with a general overview, which indicates how the *Tractatus* brings us from the world as the basis for logic and language to the world as the totality with which one has to reconcile in order to live the good life. The chapter indicates how the analysis of descriptive language makes the general form of propositions manifest, and with that ontology. It also introduces basic themes, like ostensive philosophy and descriptive essentialism, which are developed step by step in the chapters to follow.

In chapter 2, it is argued that Wittgenstein held typing to be unnecessary, and so not given prior to analysis. The insight is developed into a holism of logical space that can only be captured semi-formally. The fine-structure of the system is given, introducing basic notions such as state of things, object, and identity of object-form in a detailed way.

Chapter 3 is about projection, which is crucial to understand propositions

as logical pictures (models). After considering the pros and cons of a few local variants we arrive at a holistic notion of projection that does justice to the intensional nature of objects. Projections are allowed to vary in logical use, but per use they partition logical space in a pictorial and a non-pictorial area.

With logical space and projection in place, chapter 4 develops the notion of elementary proposition: its sign, sense and truth. Once these atoms of meaning are understood, proper expressions and material functions are introduced as abstractions over them.

The next four chapters are about propositions of finite logical complexity.

Chapter 5 begins with the tractarian view on truth-functions, and goes on to compare its three notations: truth-table signs, graphical signs and truth-operations. It is shown that truth-operations are truth-functionally complete.

Chapter 6 is about the ontological status of logical complex signs. It is a basic thought of the *Tractatus* that logical constants do not refer, but there is a clear tension between this thought and the idea that propositions are facts, and so part of ontology. To reconcile both aspects of the system, I will hold that only propositional signs show complex logical structure overtly, and that this structure is essentially a matter of form. The non-pictorial part of ontology consist of independent states of things, which allow the sense of a proposition to project logically incompatible possibilities onto them.

Chapter 7 concerns the sense of logically complex propositions against the backdrop of Frege's philosophy of language. Situations are introduced as the intensional interpretation of truth-table signs. Next it is observed that to get proper notions of contingent and logical proposition, a clear distinction must be made between the propositional sign and its symbol. By contrast to elementary sense, logically complex sense is shown to be atomistic and compositional.

The last step in developing the finite notion of proposition consists of clarifying truth. Chapter 8 combines the study of truth with that of logical consequence. This results in a formal characterization of logical propositions; i.e., a reflection on the soundness and completeness of the finite system, which in line with the philosophy is presented in a non-axiomatic way.

With the finite notion of proposition in place, chapter 9 shows how Wittgenstein has realized a perfect notation of sense. This notation vindicates speaking about *the* symbol of a proposition. It requires to discuss how the representation of sense fares with regard to different forms of substitution. The ideas are summarized in a reflection on symbol and Lindenbaum algebra's.

One way of looking at the tractarian system is that it gives Wittgenstein's solution to Russell's paradox. Chapter 10 recalls how Russell found his paradox and how he went about trying to solve it. Against this background, I discuss what Wittgenstein solution to Russell's paradox consists of. As a result, we see the system is non-reflexive; it is unable to describe its own sense conditions. Also, modern techniques to achieve reflexivity, like coding, are argued to be unavailable. Instead, Wittgenstein takes resort in an ostensive philosophy, which highlights the main features of the system to an understanding reader. The main philosophical activities are charted in a 'choreography for a swansong'.

Chapter 11 starts probing to what extent the system can be generalized into the infinite. This aspect of the system was left until after the introduction of its finite part, because trying to incorporate infinity from the start would introduce a chasm between the text and my interpretation of it. First, the reasons why the system should be infinite are given. Next, an analogy between truth-table signs and systematic analytical tableaux is used to suggest a countable system of representation. The chapter ends with a discussion on decidability, perspicuity and the independent nature of logic, to see whether the metaphysical and the human aspects of an infinitary system can be kept in balance.

Chapter 12 concerns the technical details of the infinitary system. After presenting the basic notion of a systematic sign, it is shown to what degree the infinite notation can still be regarded perfect. Next, the sense of a proposition is proved to be determinate, independent of knowing its truth value. A notion of truth is given that allows us to characterize the logical propositions from among the contingent ones. The chapter finishes with reflections on logical consequence, descriptive completeness, compactness and interpolation.

Chapter 13 is about the somewhat thorny issue of tractarian quantification. Quantifiers are abbreviations of truth-functional structure that should be considered in the context of analysis. I argue that in this context Wittgenstein's approach is correct. When considered in isolation the representations used may be ambiguous, but even then there are 'remedies' that hardly extend the system. The chapter concludes with showing how Wittgenstein's injective treatment to names and quantifiers can be had via a simple extension of the infinitary system presented in chapter 12.

Chapter 14 recapitulates the comparisons between the work of Frege, Russell, and Wittgenstein that were interspersed in the previous chapters. It concludes with giving an overview of the system as it has unfolded pleat after pleat in the remainder of the book.

Most chapters have 'interludes'. They offer historical or other points of detail without interrupting the flow of interpretation too much. Finally, there are four indices: a general one, one for persons, one for theses, and one for notation introduced.

Favorable heuristics

In hindsight it is not so hard to see that quite a few tractarian claims should be toned down. To begin with, its universality cannot be sustained. If there were a logical essence of everyday language, it would require a much richer system than an infinitary truth-functional logic based on logically independent elementary propositions. Elementary propositions can be logically dependent, as the color-exclusion problem strongly suggests, and more seems to be required than a system with truth-functional signs interpreted in an intensional space of states of things to handle opaque contexts such as 'I know that...', 'I believe that...', etc. It is also unclear whether the system is rich enough to capture all forms of extensional quantification. For example, non-first-order definable quantifiers such as 'most' seem problematic (as in: 'most books on the *Tractatus* are hard to read', meaning: there are more books on the *Tractatus* that

are hard to read than there are books on the *Tractatus* that are not hard to read). But of course I will not argue that the tractarian philosophy can be upheld in all detail. My aim is rather to show that despite its limitations, the system that the book presents has a strong, coherent interpretation. Such an interpretation should highlight what may still be of value today; e.g. it's highly original route toward ethics, or its philosophy of logic.

Early Wittgenstein was a philosopher who combined valuable logical intuitions with a disinterest in technical detail. For some the lack of logical detail may be sufficient to disregard the logical value of the *Tractatus* entirely. In the spirit of fairly recent studies – e.g., Frascolla (1994), Marion (1998), Von Kibéd (2001), Stokhof (2002), and Potter (2009) – I prefer a favorable heuristics. Instead of observing that Wittgenstein's technical remarks do not fit the mold of modern logic, it is more rewarding to ask: Is the system coherent at all, and does it give insights that have been lost in current logics? Apart from the perfect notation of sense, Wittgenstein's now obscure techniques contain gems that do not seem to have been noted before. E.g., the combination of truth-tables and truth-operations are in the spirit of tableaux; cf. Beth (1955), Smullyan (1968).

Context

To the best of my knowledge, there is no formal approach to the system in the *Tractatus* that tries to do justice to the full text. Stegmüller (1966) is an interesting model-theoretic supplement to some of the ideas in Stenius (1960). Hintikka (1986) argues that the system is basically the same as Tarski's semantics of predicate logic. Although it would surely be interesting to compare the two approaches, to do so requires developing the tractarian system in its own right. This will show that besides similarities, which depend on a quite specific interpretation, several differences between them remain; e.g., the holistic nature of elementary propositions and material functions; the use of truth-operations transforming signs; the treatment of sign and depicted as part of one logical space; the intensional interpretation of truth-table signs; the rôle of sense in the hyper-intensional system; the unique approach to quantification based on an uncommon notion of variable. . . Lokhorst (1988) is a first attempt of full coverage, but his axiomatic formalization makes heavy use of modern techniques, is hence only partly based on the text, and does not treat such key notions as operation, variable, form, or the distinction between saying and showing. Landini (2007) covers quite a few aspects of the system, but his aim is rather to rebalance the appreciation of early Wittgenstein philosophy in favor of the philosophies of Russell that had a strong influence on the *Tractatus*. Notions like operations and variables have been studied, in Sundholm (1992), Frascolla (1994), Marion (1998), Ule (2001), Potter (2000), and Von Kibéd (2001), but mostly in an informal manner or without covering other aspects of the system.

Sources

I much enjoyed using the *Kritische Edition* of McGuinness and Schulte

(Wittgenstein, 1922a). More in general, McGuinness' and Schulte's contributions to making sources and historic details available is of inestimable value.

Since I felt that the elegant translation of Pears and McGuinness (Wittgenstein, 1922b) does not always capture the sterner beauty of Wittgenstein's writing, the English translation is often my own. I did use the translations of Ramsey and Ogden and of Pears and McGuinness to check my translation for correctness. The title of the present book is from the translation of Pears and McGuinness.

I have developed the website www.tractatus.nl to facilitate research of the *Tractatus*. Based on modern web-techniques, the site enables one to view the text from different perspectives and to search it using regular expressions.

Manuscripts and typescripts are from Wittgenstein's *Nachlass* (Wittgenstein, 2000).

As to the secondary literature, apart from the now standard introductions Anscombe (1959), Stenius (1960) and Black (1964), my main influences are: Stokhof (2002), Hacker (1984), McGuinness (2001), Ishiguro (1969, 1981), Pears (1987), Von Kibéd (2001), Marion (1998), Janik and Toulmin (1973), Frascolla (1994), and Potter (2009). One may safely assume they do not agree with all that is presented here.

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My research is independent and financed entirely by myself. It is my hope that these lecture notes for imaginary students will also be of interest to some real ones.

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Index of notation

- $=$, *equisignificance*, 153
 Δ , *trace-function of logical occurrences*, 202
 Λ , *logical space*, 25
 Λ, R , *world*, 47
 Λ, π , *frame*, 57
 Ω , *operation*, 89
 $\Sigma[\varphi]$, *symbol of a proposition*, 124
 Θ , *truth-operation*, 90
 \Vdash , *enforcing in a tableau*, 208
 \Vdash , *truth in a world*, 64
 α , *state of things*, 25
 $\alpha(-)$, *material function*, 72
 α^\bullet , *non-realized state of things*, 63
 α° , *realized state of things*, 63
 $=$, *identity*
 of form, 37
 $(X_0, \dots, X_{2^n})(p_0, \dots, p_n)$,
 truth-function, 82
 $(o_0, \dots, o_n)_c$, *state of things*, 31
 \equiv , *equivalence of truth-operations*,
 157
 \mathcal{F} , *frame*, 57
 \models , *semantical consequence*, 143
 \models , *validity*, 211
 \neg , *negation (finite truth-
 function/truth-operation)*,
 80
 π , *projection*, 47
 $\pi[\Sigma[\varphi]]$, *sense of a symbol*, 205
 $\rho[X^i]$, *realization pattern*, 118
 \rightarrow , *material implication (finite
 truth-function/truth-
 operation)*,
 80
 σ , *substitution*, 151
 $\sigma[\varphi]$, *sense of a proposition*, 119
 τ , *truth-table sign*, 81
 $\tau[\chi]$, *truth-table sign of a
 proposition*, 141
 $\tau[i]$, *i-th truth-possibility*, 81
 θ_β , *truth-possibility of an open
 branch*, 203
 ε , *expression-content*, 69
 $\varphi, \psi, \chi, \dots$, *propositions*, 89
 \vdash , *logical consequence*, 141
 \vdash , *provability*, 211
 \vee , *disjunction (finite truth-
 function/truth-operation)*,
 80
 \wedge , *conjunction (finite truth-
 function/truth-operation)*,
 80
 ξ , *propositional variable*, 78
 p, q, r, s, \dots , *elementary
 proposition*, 81
 $(F)()$, *contradiction*, 125
 $(T)()$, *tautology*, 125
 $F(\varepsilon)$, *expression-form*, 70
 $F(o)$, *form of an object*, 35
 N , *truth-operation of joint denial*,
 7
 o , *object-content*, 35
 r , *reality*, 134

$S^\circ\chi$, *conjugate of a proposition*,
199

propositions, 199

$\text{Tr}[S\chi]$, *ordered tree of*

w , *world*, 47

"This is the book I always wanted to read but could not, because no-one had yet written it: an in-depth, comprehensive investigation of Wittgenstein's views on language and logic in the *Tractatus*. Jaap van der Does combines technical prowess and philosophical sensitivity, and thereby gives us a much needed addition to the many existing introductions. An invaluable companion for anyone who really wants to master Wittgenstein's early masterpiece."

Martin Stokhof, University of Amsterdam

Wittgenstein's *Tractatus Logico Philosophicus* is a highly original ethical deed that involves presenting a logical system in literary fashion. This book aims for a strong interpretation of his text, both philosophically and technically. It makes clear how Wittgenstein's early views on ethics and logic are related to his symbolic turn. The symbolic nature of contingent propositions is charted in detail, and logical propositions are characterized as empty forms. All key concepts are made as explicit as possible. The finite system is covered but also an infinite generalization that the text just hints at. It is argued that Wittgenstein anticipates the elegant tableaux methods developed much later; his treatment of quantification is shown to be correct. Still, all that is of value in life is inversely related to the system and its ontology, which purports to give a clear view on what can be said and what must be passed over in silence.

